MAT 1700

LØSNINGSFORSLAG

SEMINAR # 6

Merk: Oppgavene 3 og 4 dekker pensum gjennomgått i forelesning

Solutionsmanual Oppgavesett # 6

(Friday, Feb 22, 2008)

Oppgave 1 $U(W) = \ln(W)$

$E(W) = .20(30) + .80(5) = \underline{\underline{10}}$

$U[E(W)] \equiv$ utility from the actuarial value $= \ln(10)$

$= U[10] = \ln(10) = \underline{\underline{2.30}}$

(b) $E[U(W)] = .20\,U(30) + .80\,U(5)$

$= .20(3.40) + .80(1.61) = \underline{\underline{1.97}}$

(c) Sikkerhetsekvivalent (SE) beløp

Verdien av spillets aktuariske (forventnings-) beløp justert for usikkerhet. Med risk-aversjon, er derfor SE-beløp < $E(W)$ = aktuarisk beløp.

Altså: Det usikre beløpet, $E(W) = 10$, har SE-verdi $= 7.17$. Det er maks. beløpet vi er villig til å betale for å delta i dette spillet med forventet beløp $= 10$.

"Risk-premie" $= E(W) -$ SE-beløp $= \underline{\underline{2.83}}$

dvs. nyttefunksjonen reflekterer (indirekte) aktørens holdning til, og pris (her: 2.83) på risiko!

Oppgave 1

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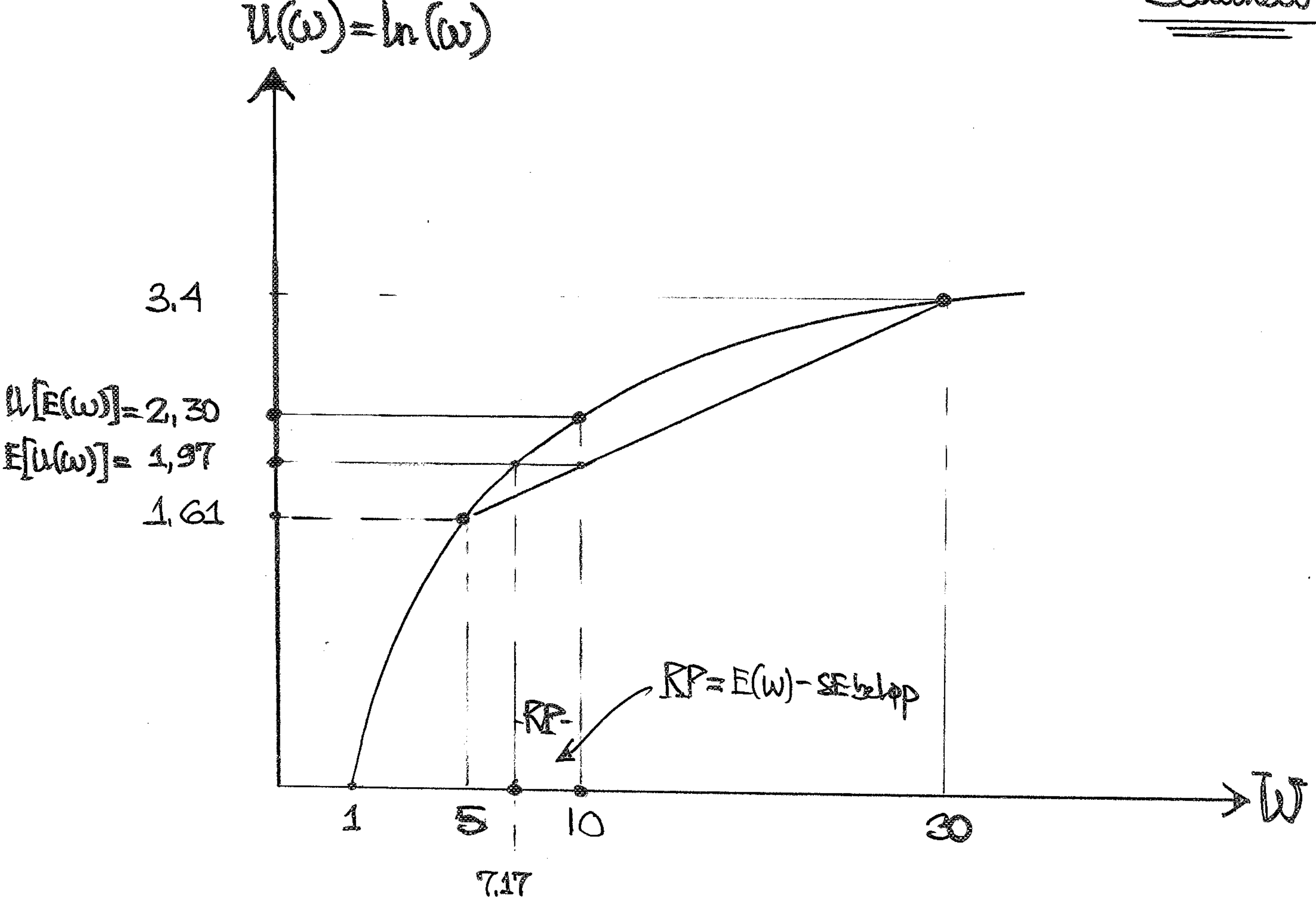

Sikkerhetsekvivalent (SE) - beløp = $\underline{7{,}17}$

(Risikopremie ≡ E(ω) − Sikkerhetsekvivalent beløp = 100 − 7,17 = $\underline{2{,}83}$)
≡ RP

$$E[U(\omega)] = p_L U(\omega_L) + p_H U(\omega_H) = .20(30) + .80(5) = 1.97$$

$$\Downarrow$$

$$W = e^{E[U(\omega)]} = e^{1{,}97} = \underline{7{,}17}$$

$U[E(\omega)] > E[U(\omega)] \Rightarrow$ risk-aversion !

2,30 > 1,97

Oppgave 2 $U(W) = W^2$

(a) $E(W) = .50(50) + .50(150) = 100$

$U[E(W)] = U(100) = 100^2 = \underline{10.000}$

$E[U(W)] = .50\,U(50) + .50\,U(150)$
$= .50(50^2) + .50(150^2) = 1250 + 11250$
$= \underline{\underline{12500}}$

Ja; fordi forventet nytte (12500 utils) er større enn nytten av spillets forventnings (aktuarske) verdi.

$U[E(W)] < E[U(W)]$
$10.000 < 12500$

(b) $E[U(W)] = 12500; \quad W = (12500)^{1/2} = \underline{\underline{111,8}}$

er maksimal <u>betalingsvillighet</u> for denne aktøren

(c) <u>Risikopremien</u> $= \underline{-11,80}$

$= E(W) -$ Sikkerhetsekvivalent beløp = SE-beløp

$= 100 - \sqrt{E[U(W)]} = 100 - \sqrt{12500}$

$= 100 - 111,80 = \underline{\underline{-11,80}}$

Oppgave 2 Risk-seeker

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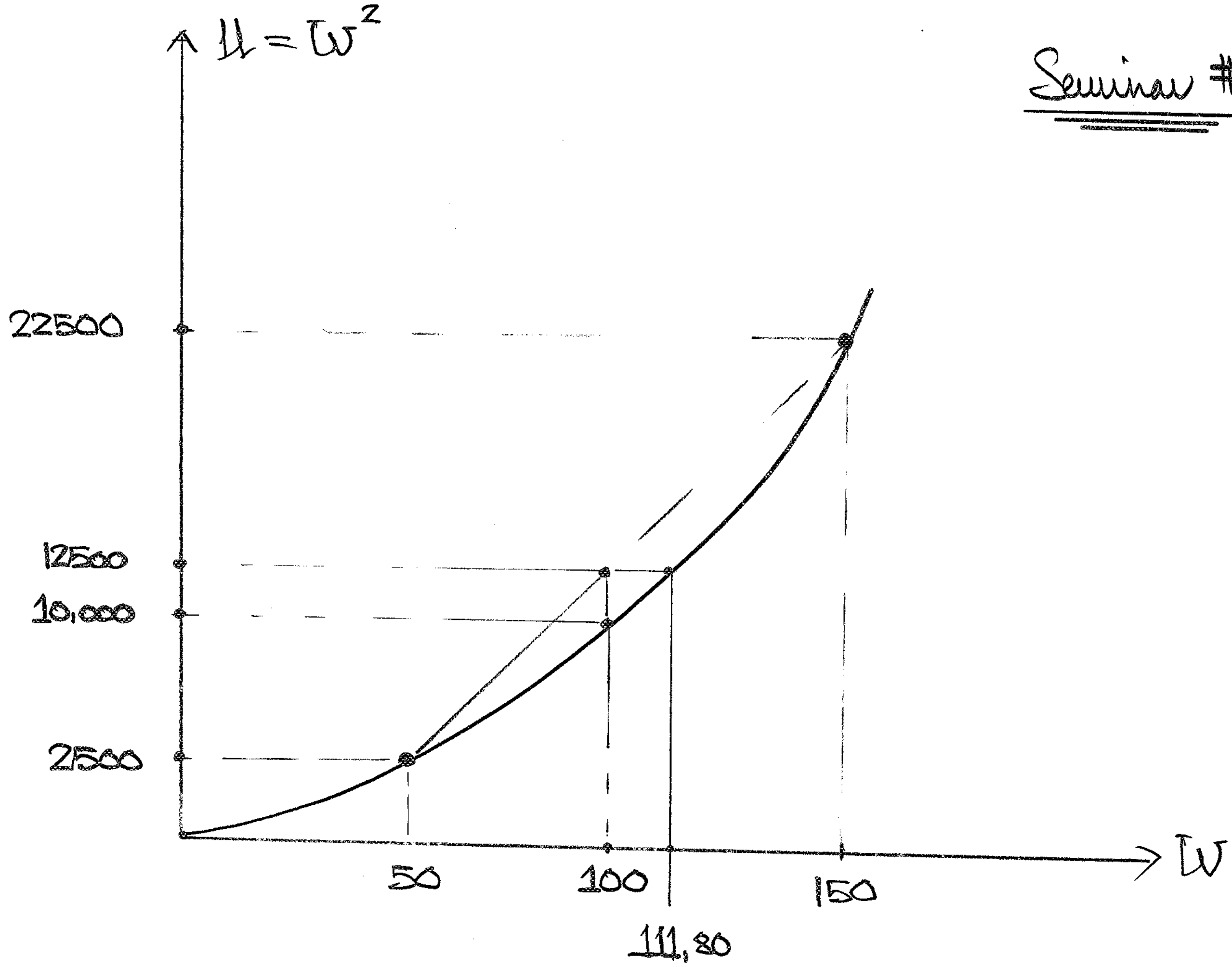


Oppgave 3

$$E(r_P) = \omega_1 E(r_1) + (1-\omega_1) E(r_2)$$

$$\sigma_P^2 = \omega_1^2 \sigma_1^2 + (1-\omega_1)^2 \sigma_2^2 + 2\omega_1 (1-\omega_1) \text{Kov}(r_1 r_2)$$

$$= \omega_1^2 \sigma_1^2 + (1-\omega_1)^2 \sigma_2^2 + 2\omega_1 \text{Kov}(r_1 r_2) - 2\omega_1^2 \text{Kov}(r_1 r_2)$$

$$\frac{\partial \sigma_P^2}{\partial \omega_1} = 2\omega_1 \sigma_1^2 + 2(1-\omega_1)\sigma_2^2 (-1) + 2\text{Kov}(r_1 r_2) - 4\omega_1 \text{Kov}(r_1 r_2)$$

$$2\omega_1 \sigma_1^2 + 2\omega_1 \sigma_2^2 - 4\omega_1 \text{Kov}(r_1 r_2) = 2\sigma_2^2 - 2\text{Kov}(r_1 r_2)$$

$$\Rightarrow \omega_1 \left(\sigma_1^2 + \sigma_2^2 - 2\text{Kov}(r_1 r_2)\right) = \sigma_2^2 - \text{Kov}(r_1 r_2)$$

$$\omega_1^* = \frac{\sigma_2^2 - \text{Kov}(r_1 r_2)}{\sigma_1^2 + \sigma_2^2 - 2\text{Kov}(r_1 r_2)} \quad ; \quad \text{Kov}(r_1 r_2) = \rho_{12}\sigma_1\sigma_2$$

$$\omega_2^* = (1 - \omega_1^*)$$

Innsett at $\text{Kov}(r_1 r_2) = \rho_{12}\sigma_1\sigma_2$; $[-1.00 \leq \rho_{ij} \leq +1.00]$

$$\omega_1^* = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

$\underline{\underline{\rho = -1}}$: $\omega_1^* = \dfrac{\sigma_2^2 + \sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2} = \dfrac{\sigma_2(\sigma_2 + \sigma_1)}{(\sigma_1 + \sigma_2)(\sigma_1 + \sigma_2)}$

perfekt, negativ korr. koeff

$$= \underline{\underline{\frac{\sigma_2}{\sigma_1 + \sigma_2}}}$$

Oppgave 3, con't

$\rho = +1$: perfekt, positiv korr.-koeff.

$$\omega_1^* = \frac{\sigma_2^2 - \sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2} = \frac{\sigma_2(\sigma_2 - \sigma_1)}{(\sigma_1 + \sigma_2)(\sigma_1 - \sigma_2)}$$

Oppgave 4 Likevektet portefølje med N aksjer

(a) Variansledd = 200 = N

Kovar-ledd = $N^2 - N = N(N-1) = 200(199)$

$= \underline{39.800}$!

$$Var(r_P) = \frac{1}{N}\sum \frac{\sigma_i^2}{N} + \left(N - \frac{1}{N}\right) \cdot \frac{\sum\sum Cov(r_i r_j)}{N(N-1)}$$

$$= \frac{1}{N}\bar{\sigma}_i^2 + \left(1 - \frac{1}{N}\right)\overline{Kov(r_i, r_j)}$$

$$= \frac{1}{200}(0,29) + \left(1 - \frac{1}{200}\right) \times 0.065 = .0015 + .0647$$

$$= \underline{0,0662}\ ; \quad \sigma = (0,0662)^{1/2} = \underline{0,2573}$$

(b) 10 tilfeldig valgte aksjer

$$Var(r_P) = 0,10(0,29) + \left(1 - \frac{1}{10}\right) 0,065$$

$$= 0,0290 + 0,0644 = \underline{0,0934}$$

$$\Rightarrow \sigma_P = \underline{0,3055} \ll 0,54 = (0,29)^{1/2}$$

Ja, dvs. virker!

Oppgave 5 (Ref. oppgave 3 i oppg.sett # 3)

$U(x,y) = 2\sqrt{x} + y$

$m = 10;\ p_x = 0{,}50$

$p_y = 1$

$MU_x = 1/\sqrt{x}$

$MU_y = 1$

$\Rightarrow \frac{MU_x}{MU_y} = \frac{p_x}{p_y} = \frac{p_x}{1}$

$\Rightarrow 1/\sqrt{x} = p_x;\ x = 1/p_x^2$

Initially; for $p_x = 0{,}50$; $x = 1/(.50)^2 = \underline{4}$

into budget: $0{,}50(4) + y = \underline{10};\ y = \underline{\underline{8}}$

$U(x,y) = U(4,8) = 2\sqrt{4} + 8 = \underline{\underline{12}}$

(a) Compensating variation (CV) due to price-reduction of x ≡ chocolate consumption/demand

$x = 1/(.20)^2 = \underline{\underline{25}};\ U(25,y) = 2\sqrt{25} + y = \underline{12}$

$y = \underline{2}$ ("basket B")

Budget: $0{,}20(25) + 2 = 7 < 10 \Rightarrow \underline{\underline{CV = -3}}$

"New basket" C: $0{,}20(25) + y = 10;\ \underline{\underline{y = 5}}$

$U(25,5)_{\text{basket C}} = 2\sqrt{25} + 5 = \underline{\underline{15}}$

(b) Equivalent variation (zero income effect) ≡ EV

$U(x,y) = 15$

$p_x = 0{,}50;\ x = 4$

$U(x,y) = 15 = 2\sqrt{4} + y = 15;\ y = 11$

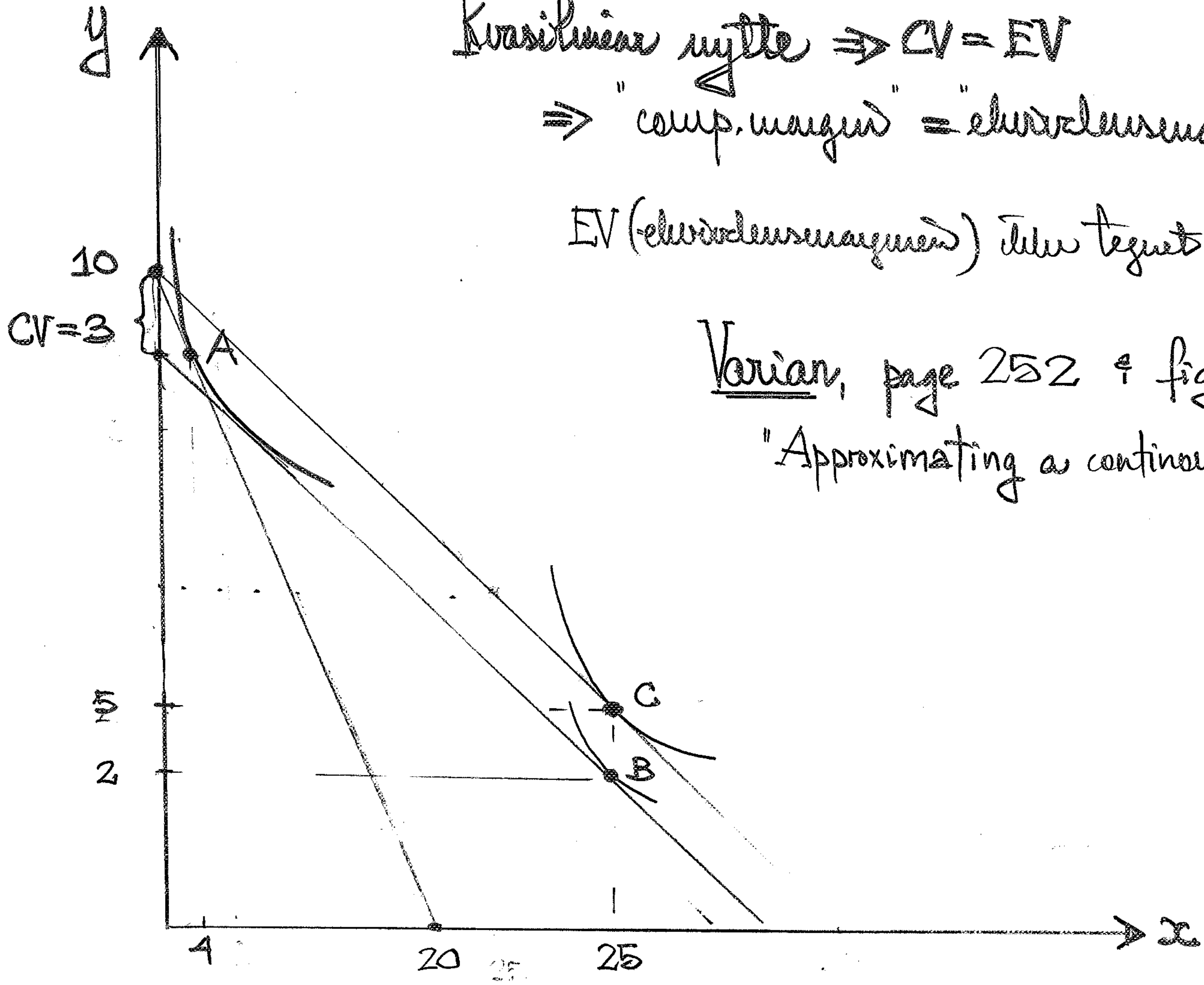
Budget: $0{,}50(4) + 11 = 13 > 10;\ \underline{\underline{EV = 13 - 10 = 3}}$

Oppgave 5, con't

Kvasilineær nytte ⇒ CV = EV
⇒ "comp. margin" = "ekvivalensmarginen"

EV (ekvivalensmarginen) ikke tegnet inn!

Varian, page 252 & fig. 14.2
"Approximating a continous demand"



Varian, pp. 254 - 258:
Compensating and Equivalent Variation

Seminar #6

Oppgave 6

Seminar #6

Ref. oppgave 2 i Seminar #3

$U(x,y) = x \cdot y$

$x \equiv$ demand for food; $p_x = 9$ (initially)

$y \equiv$ — " — clothing; $p_y = 1$

$m \equiv$ monetary (cash) income $= 72$

Initial optimal consumption (demand) requires that;

① on budget-line: $9x + y = 72$

② $\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = \frac{9}{1} \Rightarrow y = 9x$

Two equations in two unknowns;

$$\left.\begin{aligned} 9x + y &= 72 \\ y &= 9x \end{aligned}\right\} \Rightarrow 9x + 9x = 72;\ \underline{x = 4};\ \underline{y = 36}$$

$U(x,y) = x \cdot y = 4 \cdot 36 = \underline{144}$

Let p_x fall to 4 (from initially 9) per unit... new consumption?

$$\left.\begin{aligned} 4x + y &= 72 \\ y &= 4x \end{aligned}\right\} \Rightarrow 4x + 4x = 72;\ \underline{x = 9};\ \underline{y = 4(9) = 36}$$

$U(x,y) = 9 \cdot 36 = \underline{324}$ (Finale consumption-choice)

(a) Compensating variation (CV) og Equivalent variation (EV)

CV: Holding utility unchanged from initial basket;

$U(xy) = x \cdot y = 4 \cdot 36 = \underline{144}$

$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = 4;\ y = 4x$

"Decomposition basket"

$U(x \cdot y) = 144 = x \cdot y = x(4x) = 4x^2;\ \underbrace{x = 6;\ y = 24}$

Budget: $p_x \cdot x + p_y \cdot y = 4(6) + 1(24) = \underline{48}$

$\underline{CV} = (m - \text{'new budget'}) = 72 - 48 = \underline{24}$ ... amount of cash that can be given up after price-reduction in order to be as well off as before price-reduction

Oppgave 6, fortsettelse (Ref. oppgave 2 - Seminar #3)

(a) CV = 24; but what's the EV?

EV: Holding utility unchanged from finale consumption-basket
$U(x \cdot y) = x \cdot y = 9 \cdot 36 = \underline{324}$

Evaluating the 'decomposition-basket' ($x = 6$; $y = 24$) at initial price ($p_x = 9$);

$U(x \cdot y) = 324 = x \cdot y = x(9x)$; $\underline{x = 6}$; $\underline{y = 54}$

Budget: $9(6) + 1(54) = 108 > m = 72$

$\underline{EV} = 108 - 72 = \underline{\underline{36}}$ ... amount of cash received before price-reduction in order to keep utility the same ($U(x \cdot y) = 324$) as after price-reduction

(b) CV ≠ EV since income-effect ≠ 0

(see problem 2 - Seminar #3)

Varian; pp. 251-2 and pp. 102-103 Varian

Quasi-linear preferences (utility): Zero 'income-effect' since changes in income don't affect demand. Thus; CV = EV.
In general, for non-quasi linear preferences; there is an income-effect present. Thus; demand for a good changes when income changes, and CV ≠ EV (ie. change in consumer surplus represents only an approximization to the change in consumers' utility. See fig. 6.8: "Quasilinear preferences" page 103 Varian - vertical Engle-curve;

"Zero income effect"
"... as income changes, the demand for good 1 remains constant."